ÉRETTSÉGI VIZSGA ● 2012. október 16.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors and incomplete solutions in the conventional way.
2. The first one of the grey rectangles under each problem shows the maximum attainable score on that problem. The **points given by the examiner** are to be entered in the rectangle next to that.
3. **If the solution is perfect,** it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.
5. Do not assess anything except diagrams that is **written in pencil**.

Assessment of content:

1. The markscheme may contain more than one solution to some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error occurs. If the reasoning remains correct, with the error carried forward, and the nature of the task does not change, then the points for the rest of the solution should be awarded.
4. In the case of a **principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or to the next part of the problem and is used correctly there, the maximum score is due for the next part, provided that the error has not changed the nature of the task to be completed.
5. Where the markscheme shows a remark or unit **in brackets**, the solution should be considered complete without that remark or unit as well.
6. If there are **more than one different approaches** to a problem, assess only the one indicated by the candidate.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
9. **Assess only four out of the five problems in Section II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I

1. a)		
The income is $5\cdot10^6\cdot200\ \left(=10^9\right)$ (forints).	1 point	
The total prize paid to the winners: $4\cdot10^7+2\cdot10^6+8\cdot10^6+1{,}5\cdot10^8+2\cdot10^8+2\cdot10^8$ $(=6\cdot10^8)$ (forints)	1 point	
So the difference is 400 million forints.	1 point	
Total:	**3 points**	

1. b)		
(The chance of drawing any particular ticket is equal.) The number of favourable cases is 550 844,	2 point	*1 point may be awarded if it is clear that the candidate added the right numbers but made a calculation error.*
thus the probability in question is $p=\frac{550844}{5\cdot10^6}\approx0.11$.	2 point	*0.1 is also accepted. A correct answer given as a percentage is also accepted.*
Total:	**4 points**	

1. c) solution 1		
The expected value of the prize won is $\frac{4\cdot10^7+2\cdot10^6+8\cdot10^6+1{,}5\cdot10^8+2\cdot10^8+2\cdot10^8}{5\cdot10^6}=$	2 points	*Do not divide.*
= 120 (forints).	1 point	
The expected value of the profit is therefore (120 – 200 =) –80 forints.	1 point	
Total:	**4 points**	

1. c) solution 2		
The amount gained by the company that issued the lottery tickets equals the negative of the total profit made by the players.	2 points	
Therefore the expectance of the profit made by one player is $-\frac{400000000}{5000000}=$	1 point	
= –80 forints.	1 point	
Total:	**4 points**	

1. c) solution 3

prize	profit	probability		
10 000 000	9 999 800	$\frac{4}{5\cdot 10^6}(=0.0000008)$	2 points	*1 point if there is one error, no points for more than one error.*
50 000	49 800	$\frac{40}{5\cdot 10^6}(=0.000008)$		
10 000	9 800	$\frac{800}{5\cdot 10^6}(=0.00016)$		
1 000	800	$\frac{150000}{5\cdot 10^6}(=0.03)$		
500	300	$\frac{400000}{5\cdot 10^6}(=0.08)$		
200	0	$\frac{10^6}{5\cdot 10^6}(=0.2)$		
0	–200	$\frac{3\,449\,156}{5\cdot 10^6}(=0.6898312)$		

The expected value can also be obtained from the formula $E(X)=\sum_i x_i \cdot p_i$.	1 point	*This point is also due if the reasoning is only reflected by the solution.*
The expected value of the profit is –80 Ft.	1 point	
Total:	**4 points**	

2. solution 1

(Let x denote the number that is decreased by 15, and let y denote the other number.) $\left.\begin{aligned} x+y&=29 \\ (x-15)(y+15)&=5xy \end{aligned}\right\}$	2 points	*Do not divide.*
(For example,) y expressed from the first equation and substituted in the second equation: $(x-15)(44-x)=5x(29-x)$.	1 point	*If x is expressed:* $(14-y)(y+15)=$ $=5(29-y)y$.
The operations carried out: $-x^2+59x-660=145x-5x^2$.	3 points	*1 point for eliminating the brackets on the left-hand side, 1 point for adding like terms, 1 point for eliminating the brackets on the right-hand side.*
Rearranged: $4x^2-86x-660=0$. (Simplified: $2x^2-43x-330=0$.)	1 point	*If a quadratic equation in y is set up:* $2y^2-73y+105=0$.
The solutions of the equation are –6 and 27.5.	2 points	

If the number to be decreased by 15 is –6 then the other number is 35.	1 point	
If the number to be decreased by 15 is 27.5 then the other number is 1.5.	1 point	
Checking against the wording of the problem: If the two numbers are –6 and 35 then (their sum is 29 and) their product is –210. The numbers obtained by changing the factors are –21 and 50, their product is –1050, which is really 5 times –210.	1 point	*1 point may be awarded if the candidate states that the solutions are found valid by checking against the wording, but no calculation is shown.*
If the two numbers are 27.5 and 1.5 then (their sum is 29 and) their product is 41.25. The numbers obtained by changing the factors are 12.5 and 16.5, their product is 206.25, which is really 5 times 41.25.	1 point	
Total:	**13 points**	

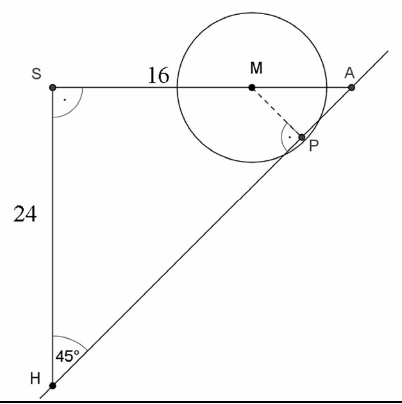
2. **solution 2**		
(If x is the number to be increased by 15 then the other number is $29 - x$. Let c denote the product of the two numbers.) $\left.\begin{aligned} x(29-x) &= c \\ (x+15)(14-x) &= 5c \end{aligned}\right\}$	2 points	*Do not divide.*
Subtracting the first equation from the second one: $-30x + 210 = 4c$.	1 point	
Hence expressing x: $x = 7 - \frac{2c}{15}$.	1 point	*If c is expressed:* $c = -\frac{15}{2}x + \frac{105}{2}$.
Substituted back in the first equation: $\left(7 - \frac{2c}{15}\right)\cdot\left(22 + \frac{2c}{15}\right) = c$.	1 point	$x(29-x) = -\frac{15}{2}x + \frac{105}{2}$
The operations carried out: $154 - 2c - \frac{4c^2}{225} = c$.	1 point	
Multiplied by 225 and rearranged: $0 = 4c^2 + 675c - 34\,650$.	1 point	$2x^2 - 73x + 105 = 0$
The roots of the equation are –210 and 41.25.	2 points	
From the first root, $x = 35$, Then the other number is –6.	1 point	
From the second solution, $x = 1.5$, then the other number is 27.5.	1 point	
Checking: see **solution 1** above.	2 points	
Total:	**13 points**	

3. a)		
(Because of the square root,) $x \geq 0$,	1 point	
(because of the logarithm,) $\sqrt{x} > 0$.	1 point	
The set in question: $]0;+\infty[$.	1 point	*The answer x > 0 is also accepted.*
Total:	**3 points**	

3. b)		
(Because of the logarithm,) cos $x > 0$,	1 point	
and (because of the square root,) $\log_2(\cos x) \geq 0$,	1 point	
that is, $\cos x \geq 1$.	1 point	
(Because of the range of the cosine function,) $\cos x = 1$.	1 point	
The domain is $\{x \in \mathbf{R} \mid x = k \cdot 2\pi, k \in \mathbf{Z}\}$.	1 point	*$x = k \cdot 2\pi$, $k \in \mathbf{Z}$ is also accepted. Do not award this point if $k \in \mathbf{Z}$ is not stated.*
Total:	**5 points**	

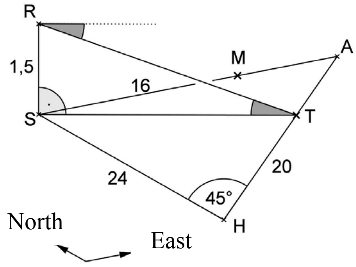
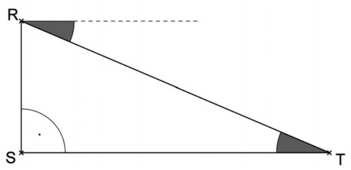
3. c)		
(Because of the base of the logarithm,) $x > 0$ and $x \neq 1$.	1 point	*This point is only due if both conditions are listed.*
(Because of the logarithm,) $\cos^2 x > 0$,	1 point	
thus $\cos x \neq 0$, that is,	1 point	
$x \neq \frac{\pi}{2} + k \cdot \pi$, where $k \in \mathbf{Z}$.	1 point	
The domain is $\mathbf{R}^+ \setminus \left(\{1\} \cup \left\{ \frac{\pi}{2} + k \cdot \pi \right\} \right)$, where $k \in \mathbf{N}$.	1 point	*$k \in \mathbf{Z}$ instead of $k \in \mathbf{N}$ is also accepted. The answer x > 0 but $x \neq 1$ and $x \neq \frac{\pi}{2} + k \cdot \pi$ ($k \in \mathbf{Z}$) is also acceptable.*
Total:	**5 points**	

Remark. Award at most 4 points if there is no statement of the possible values of k at all.

4. a) solution 1		
Correct diagram showing the conditions of the problem. *S* denotes the island, *M* denotes the lifeboat and *H* denotes the ocean liner. Let *A* denote the intersection of the path of the ocean liner with the line *SM*. 	2 points	*These 2 points are also due in the case of a correct solution without a diagram.*
Triangle *HSA* is right-angled and isosceles, therefore $AS = 24$ (km), so	1 point	*This point is also due if the idea is reflected by the diagram.*
$MA = 8$ (km).	1 point	
Triangle *APM* is right angled and has an angle of 45º (therefore isosceles),	1 point	*This point is also due if the idea is reflected by the diagram.*
so $MP = 4\sqrt{2}\ (\approx 5.7)$ (km).	1 point	
(Since $MP < 6$ km,) it follows that the crew of the liner may detect the distress signals.	1 point	
Total:	**7 points**	

4. a) solution 2		
Correct diagram showing the conditions of the problem. (see with **solution 1**).	2 points	*These 2 points are also due in the case of a correct solution without a diagram.*
In triangle *SHM*, $\tan\sphericalangle SHM = \dfrac{16}{24}$, hence $\sphericalangle SHM \approx 33.7°$.	1 point	
$\sphericalangle MHP \approx 45° - 33.7° = 11.3°$	1 point	
$MH = \sqrt{16^2+24^2} \approx 28.8$ (km)	1 point	
$\sin 11.3° \approx \dfrac{MP}{28.8}$, thus $MP \approx 5.7$ (km).	1 point	
(Since $MP < 6$ km,) it follows that the crew of the liner may detect the distress signals.	1 point	
Total:	**7 points**	

4. a) **solution 3**		
Correct diagram in the coordinate plane, showing the conditions of the problem. Let the island S be the origin, let the horizontal axis point eastwards and let the vertical axis point northwards. Let one unit be 1 km.	2 points	*These 2 points are also due in the case of a correct solution without a diagram.*
The equation of line *HA*, the path of the ocean liner is $y = x - 24$.	1 point	
The distress signals are detectable within a circle of radius 6 centred at the point (16; 0). The equation of this circle is $(x - 16)^2 + y^2 = 36$.	1 point	*According to the distance formula of point and line, the distance of point (16; 0) from the line* $x - y - 24 = 0$ *is* $d = \frac{\lvert 16-0-24 \rvert}{\sqrt{1^2+1^2}} =$ $= 4\sqrt{2} < 6$.
y expressed from the equation of the line and substituted in the equation of the circle: $(x - 16)^2 + (x - 24)^2 = 36$, that is, $x^2 - 40x + 398 = 0$.	1 point	
The discriminant of this quadratic is positive (+8), so the line and the circle intersect each other.	1 point	
Therefore the crew of the liner may detect the distress signals.	1 point	
Total:	**7 points**	

4. b)		
Correct diagram showing the conditions of the problem. The aeroplane (R), the island (S), and the position of the liner (T) are the vertices of a right-angled triangle. 	1 point	*This point is also due in the case of a correct solution without a diagram.*
(The distance ST can be calculated from the cosine rule.) $ST^2 = 24^2 + 20^2 - 2 \cdot 24 \cdot 20 \cdot \cos 45°$	2 points	
$ST \approx 17.2$ (km).	1 point	*This point is also due if the candidate does not calculate the distance ST but calculates the angle of depression correctly.*
 The angle of depression equals the angle RTS of the right-angled triangle TRS (alternate angles).	1 point	*This point is due for either a diagram or the statement.*
$\tan \sphericalangle RTS = \frac{RS}{TS} \left(\approx \frac{1.5}{17.2} \right)$	1 point	
The measure of the angle of depression is ≈ 5°.	1 point	*Award no point for the answer if the candidate does not round the answer as instructed.*
Total:	**7 points**	

Remark. Award at most 6 points for this part if the answer is the complement of the angle of depression.

II

5. a)		
The suitable lines are obtained by connecting any of the 5 points marked on line *e* to any of the 4 points marked on line *f*.	1 point	
Thus the number of suitable lines is $5 \cdot 4 = 20$.	1 point	
Any suitable triangle has two vertices on one line and the third vertex on the other line.	1 point*	*This point is also due if the reasoning is only reflected by the solution.*
If the triangle has two vertices on line *e*, then these two vertices may be selected in $\binom{5}{2}$ ways,	1 point*	
thus the number of such triangles is $(10 \cdot 4 =)\ 40$.	1 point*	
If the triangle has two vertices on line *f*, then these two vertices may be selected in $\binom{4}{2}$ ways,	1 point*	
thus the number of such triangles is $(6 \cdot 5 =)\ 30$.	1 point*	
The number of suitable triangles is 40 + 30 = 70.	1 point*	
Any suitable quadrilateral has two vertices on line *e* and two vertices on line *f*.	1 point	*This point is also due if the reasoning is only reflected by the solution.*
The two points on line *e* may be selected in $\binom{5}{2}$, different ways, and the two points on line *f* in $\binom{4}{2}$ different ways.	1 point	
(Since any two vertices on line *e* may occur with any two vertices on line *f*,) the number of suitable quadrilaterals is $\binom{5}{2} \cdot \binom{4}{2} = 60$.	1 point	
Total:	**11 points**	

*The 6 points marked with * may also be awarded for the following reasoning:*

There are $\binom{9}{3}$ ways to select 3 points out of 9.	1 point	
There is no triangle formed if the three points lie on the same line.	2 points	*These 2 points are also due if it is only reflected by the solution that the complement is considered.*

Three such points can be selected in $\binom{5}{3}$, ways (line e) and $\binom{4}{3}$ ways (line f).	1 point	
There is no triangle in $\binom{5}{3}+\binom{4}{3}$ (=14) cases.	1 point	
Thus the number of triangles is $\binom{9}{3}-\binom{5}{3}-\binom{4}{3}=70$.	1 point	

5. b) solution 1		
The number of equally probable colourings is 2^9.	2 points	*Do not divide.*
For each of the two lines, there are two ways to colour every point identically,	1 point	*Award 1 point only if those cases are not considered when the points on different lines have different colours.*
thus there are 4 "favourable" colourings.	1 point	
Therefore the probability in question is $\frac{4}{2^9}\left(=\frac{1}{2^7}\approx 0.0078\right)$.	1 point	
Total:	**5 points**	

5. b) solution 2		
The colour of the first point on line e is arbitrary, and the 4 other points need to be coloured in the same way. The probability of this is $\frac{1}{2}$, for each individual point,	1 point	
which makes $\frac{1}{2^4}$.	1 point	
Analogously, the probability of the points on line f to get the same colour is $\frac{1}{2^3}$.	1 point	
(Since the appropriate colouring on line e and on line f are independent events,) the probability in question is the product of these two results,	1 point	*This point is also due if the reasoning is only reflected by the solution.*
that is, $\frac{1}{2^7}(\approx 0.0078)$.	1 point	
Total:	**5 points**	

6. a)		
The distances covered in the successive hours (in km) are successive terms of a geometric progression,	1 point	*These points are also due if the reasoning is only reflected by the solution.*
in which the first term is 45 and the common ratio is 0.955.	1 point	
$a_{10} = a_1 \cdot q^9 = 45 \cdot 0.955^9$ (≈ 29.733)	1 point	
The distance covered by the Hungarian car in the 10th hour, rounded to the nearest integer, is 30 km.	1 point	
Total:	**4 points**	

6. b)		
It is not worth replacing the battery while the inequality $45 \cdot 0.955^{n-1} \geq 20$ holds ($n \in \mathbf{N}$ and $n > 1$).	1 point	
Since the decimal logarithm function is strictly increasing,	1 point	
$(n-1)\lg 0.955 \geq \lg \frac{20}{45}$.	1 point	
Because of $\lg 0.955 < 0$, it follows that	1 point	*This point is also due if the reasoning is only reflected by the solution.*
$n \leq \frac{\lg \frac{20}{45}}{\lg 0.955} + 1 \approx 18.61$.	1 point	
(In the 18th hour it is still true that the car covers at least 20 km, but it is not true any more in the 19th hour.) The earliest hour to replace a battery is the 19th hour.	1 point	
Total:	**6 points**	

Remarks.

1. The 4 points and the 6 points are also due if the candidate correctly calculates (possibly with reasonable roundings) the distances covered in each hour, and arrives at the correct answer.

2. Award at most 4 points if the candidate solves an equation instead of an inequality in part b) and does not explain how the solution of the inequality follows from the solution of the equation (1 point for setting up the equation, 2 points for solving the equation, 1 point for the answer).

6. c)		
If n is the number of whole hours elapsed since the start of the race, then the total distance covered by the Hungarian car (without battery replacement) is the sum of the first n terms of the geometric progression:	1 point	*This point is also due if the reasoning is only reflected by the solution.*
$S_n = \dfrac{45\cdot(0.955^n-1)}{0.955-1}$.	1 point	
The task is to solve the inequality $\dfrac{45\cdot(0.955^n-1)}{0.955-1} > 1100$ (on the set of positive integers).	1 point*	
Rearranged: $0.955^n < -0.1$.	1 point*	
This inequality has no solution (because $0.955^n > 0$ for all $n \in \mathbb{N}^+$),	1 point*	
therefore the Hungarian car cannot beat the world record.	1 point*	
Total:	**6 points**	

Remark. Award at most 4 points if the candidate solves an equation instead of an inequality and does not explain how the solution of the inequality follows from the solution of the equation.

*The 4 points marked with * may also be awarded for the following reasoning:*

The sequence $\{S_n\} = \left\{\dfrac{45\cdot(0.955^n-1)}{0.955-1}\right\}$ is strictly increasing,	1 point	
and its limit is $\dfrac{45}{0.045} = 1000$,	2 points	
therefore the Hungarian car cannot beat the world record.	1 point	

7. a)		
Let the length of the base edge of the baking tin be x cm. (The volume of the tin is 4000 cm^3, so) $4000 = x^2 \cdot 6.4$.	1 point	
$x = 25$	1 point	
The surface area to be coated with enamel is $(625 + 4 \cdot 25 \cdot 6.4 =)\ 1265\,cm^2$.	1 point	
Total:	**3 points**	

7. b) solution 1		
If the height of the tin is m cm, then $4000 = x^2 m$, and the surface area to be coated with enamel (in cm^2) is $T = x^2 + 4xm$.	1 point	
With m expressed from the volume and substituted in T: $T = x^2 + \frac{16000}{x}$.	1 point	
Consider the function $T : \mathbf{R}^+ \to \mathbf{R};\ T(x) = x^2 + \frac{16000}{x}$.	1 point	*This point is due if there is any correct reference to the domain (e.g. x > 0).*
T may have a minimum where its derivative is 0.	1 point	*This point is also due if the reasoning is only reflected by the solution.*
$T'(x) = 2x - \frac{16000}{x^2}$	1 point	
$T'(x) = 0 \Leftrightarrow x^3 = 8000 \Leftrightarrow x = 20$.	1 point	
Since $T''(x) = 2 + \frac{32000}{x^3}$ is positive at $x = 20$,	1 point	*These 2 points are also due if the explanation is based on the sign change of the first derivative.*
the function T has an (absolute) minimum at $x = 20$.	1 point	
The base edge of the tins manufactured is 20 cm.	1 point	
Total:	**9 points**	

7. b) **solution 2**		
If the height of the tin is m cm, then $4000 = x^2 m$, and the surface area to be coated with enamel (in cm^2) is $T = x^2 + 4xm$.	1 point	
With m expressed from the volume and substituted in T: $T = x^2 + \frac{16000}{x}$.	1 point	
$x^2 + \frac{16000}{x} = x^2 + \frac{8000}{x} + \frac{8000}{x}$, where $x > 0$.	1 point	
Applying the inequality of the arithmetic and geometric means to the three terms of the sum on the right-hand side: $x^2 + \frac{8000}{x} + \frac{8000}{x} \geq 3 \cdot \sqrt[3]{x^2 \cdot \frac{8000}{x} \cdot \frac{8000}{x}} =$ $= 3 \cdot \sqrt[3]{64 \cdot 10^6} = 1200$.	2 points	
Thus the surface area to be coated with enamel cannot be smaller than 1200 cm^2.	1 point	
Equality is possible in the case when $x^2 = \frac{8000}{x}$, that is, for $x^3 = 8000$.	1 point	
Hence $x = 20$.	1 point	
Therefore the base edge of the tins manufactured is 20 cm.	1 point	
Total:	**9 points**	

7. c)		
If a tin is selected at random, the probability of its being faulty is 0.02, so the probability that it is good is 0.98.	1 point	*These 2 points are also due if the reasoning is only reflected by the solution.*
The probability in question can be calculated from the binomial distribution:	1 point	
$P(2 \text{ faulty}) = \binom{50}{2} \cdot 0.02^2 \cdot 0.98^{48} \approx$	1 point	
≈ 0.186.	1 point	
Total:	**4 points**	

8. solution 1		
From the cosine rule for side AC of triangle ABC: $AC^2 = AB^2 + BC^2 - 2\cdot AB\cdot BC\cdot 0.5$.	2 points	
$AB^2 = 50$	1 point	
$BC^2 = 16 + (p+4)^2 =$	1 point	
$= p^2 + 8p + 32$	1 point	
$AC^2 = 81 + (p-1)^2 =$	1 point	
$= p^2 - 2p + 82$	1 point	
These result substituted in the expression of the cosine rule: $p^2 - 2p + 82 = p^2 + 8p + 82 - \sqrt{50}\cdot\sqrt{p^2+8p+32}$.	1 point	
Rearranged: $\sqrt{50(p^2+8p+32)} = 10p$.	2 points	
Since there is a positive number on the left-hand side, it follows that $p > 0$.	1 point	*This point may also be awarded if the candidate rejects the negative root of the quadratic obtained.*
Squared and simplified: $p^2 + 8p + 32 = 2p^2$,	1 point	
hence $p^2 - 8p - 32 = 0$.	1 point	
The roots of this equation are $p_1 = 4 + 4\sqrt{3}$, $p_2 = 4 - 4\sqrt{3}$.	2 points	*Award at most 2 points on this part if approximate values are calculated.*
(Since $p > 0$,) $p = 4 + 4\sqrt{3}$.	1 point	
Total:	**16 points**	

8. solution 2		
The angle enclosed by the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ is 60°, so their scalar product is $\overrightarrow{BA}\cdot\overrightarrow{BC} = \lvert\overrightarrow{BA}\rvert\cdot\lvert\overrightarrow{BC}\rvert\cdot\cos 60^\circ =$	1 point	
$= \dfrac{AB\cdot BC}{2}$,	1 point	
where $AB = \sqrt{(-5)^2 + 5^2} = 5\sqrt{2}$,	1 point	
and $BC = \sqrt{4^2 + (p+4)^2}$.	1 point	
The scalar product can also be expressed as the sum of the products of the corresponding coordinates.	1 point	*This point is also due if the reasoning is only reflected by the solution.*
Since $\overrightarrow{BA}(-5;5)$ and $\overrightarrow{BC}(4;p+4)$,	1 point	
$\overrightarrow{BA}\cdot\overrightarrow{BC} = -20 + 5\cdot(p+4) = 5p$.	2 points	

The two expressions of the scalar product must be equal, therefore	1 point	*This point is also due if the reasoning is only reflected by the solution.*
$\dfrac{5\sqrt{2}\cdot\sqrt{16+(p+4)^2}}{2}=5p$.	1 point	
Since the left-hand side of the equation is positive, it follows that $p > 0$.	1 point	*This point may also be awarded if the candidate rejects the negative root of the quadratic obtained.*
Rearranged, squared and further rearranged: $p^2-8p-32=0$.	2 points	
The roots of this equation are $p_1=4+4\sqrt{3}$, $p_2=4-4\sqrt{3}$.	2 points	*Award at most 2 points on this part if approximate values are calculated.*
(Since $p > 0$,) $p=4+4\sqrt{3}$.	1 point	
Total:	**16 points**	

8. **solution 3**		
The gradient of line *AB* is $\dfrac{1-(-4)}{2-7}=-1$,	2 points	*Do not divide.*
Thus it encloses an angle of 135° (–45°) with the positive *x*-axis.	1 point	
Two angles of the triangle bounded by *x*-axis and the lines *AB* and *BC* are 45° and 60°,	2 points	
so the third angle is 75°.	1 point	
It follows that line *BC* encloses an angle of 75° with the positive *x*-axis,	2 points	
therefore its slope is tan 75°.	1 point	
The exact value of this (e.g. from the appropriate addition formula) is $\tan 75^\circ = \left(\tan(45^\circ+30^\circ)=\dfrac{\tan 45^\circ+\tan 30^\circ}{1-\tan 45^\circ\cdot\tan 30^\circ}=\right)2+\sqrt{3}$.	2 points*	
The equation of line *BC* is $y+4=(2+\sqrt{3})(x-7)$.	2 points	
The coordinates of point *C* satisfy this equation,	1 point	*This point is also due if the reasoning is only reflected by the solution.*
thus $p+4=(2+\sqrt{3})(11-7)$,	1 point	
and hence $p=4+4\sqrt{3}$.	1 point*	
Total:	**16 points**	

*Remark. If the candidate calculates with an approximate value of tan 75°, and thus obtains an approximate value for p, the 3 points marked with * are not awarded (tan 75° ≈ 3.732, p ≈ 10.928).*

9. a)		
Statement (1) is false.	1 point	
A simple graph on 5 points may have at most 10 edges, so it cannot have 11.	2 points	
Statement (2) is true.	1 point	
(The degree of each vertex is at most 4.) If the degree of each vertex were 3, then the sum of the degrees in the graph would be odd (15), which is impossible.	2 points	
Total:	**6 points**	

9. b)		
If a complete graph on four points, and an isolated point is obtained by colouring 6 edges, then the resulting graph is not connected, that is, it meets the conditions.	2 points	
It is not possible to get a non-connected graph in any other way, since two (not necessarily connected) components with 2 and 3 vertices could have at most $1 + 3 = 4$ edges altogether.	2 points	
In the case of a complete graph on four points, the isolated points can be selected in 5 ways, and that determines the 6 edges to colour. Thus the number of such graphs is 5.	2 points	
The complete graph on five points has 10 edges.	1 point	
Hence the number of ways to select 6 edges to be coloured is $\binom{10}{6}$ (= 210).	2 points	
Therefore the probability in question is $p = \frac{5}{210}$ (≈ 0.024).	1 point	
Total:	**10 points**	